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**Real-Time Detection of Transient Signals  
Using Spline-Wavelets**

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# REAL-TIME DETECTION OF TRANSIENT SIGNALS USING SPLINE-WAVELETS

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## Abstract

Compactly supported spline-wavelets are used to process simulated transient signals in a noisy environment. It is found that linear-phase spline-wavelets can be effectively used as near-optimal time-domain windows to localize the signal onsets. This technique does not require the a priori knowledge of the signal. Results are demonstrated numerically by the detection of signal arrivals in noise.

## 1. Introduction

The arrival-time and spectrum of a transient signal contain essential information about the position and velocity of a moving target in the Pulse Doppler mode of radar operation. Conventional techniques determine the arrival-times of the echoes by either applying a threshold algorithm or using statistical methods after noise is removed from the signal. In this paper, we apply the decomposition algorithm of the wavelet technique to determine the time of arrival of a class of nonstationary signals of which the arrival times and frequency spectra are both unknown.

Traditional Fourier transform method fails to determine the arrival-time since it produces only spectral-domain information. Time-domain techniques without time and frequency localization properties are ineffective for this type of problems. The short-time Fourier Transform (STFT) generates time localization in the Fourier method by inserting a window function in the Fourier transform.

The choice of the window function is critical to its success. For high frequency transient signals, the time-window must be very narrow to produce good time-domain localization. On the other hand, a wide window is needed to deduce the spectral components of a low-frequency transient. Because STFT uses a fixed window for all frequencies, it is not effective for wideband signals. The integral wavelet transform, with its zoom-in and zoom-out capability, can be used in a wide variety of signal processing applications [1]. In this paper, we apply the compactly supported spline-wavelet developed by Chui and Wang [2] for the detection of arrival-times of transient signals.

## 2. Wavelet series and integral wavelet transform

Analogous to the Fourier analysis, it has been shown that any finite energy signal  $f(t)$  can be represented by a wavelet series [3]

$$(1) \quad f(t) = \sum d_{k,j} \psi_{k,j}(t)$$

where

$$(2) \quad \psi_{k,j}(t) = 2^{k/2} \psi(2^k t - j)$$

is the basic wavelet  $\psi$  dilated by a factor  $2^k$  and translated by  $j$  on the real line. The coefficients are given by the integral wavelet transform (IWT) defined by

$$(3) \quad (W_{\tilde{\psi}} f)(\beta, \alpha) = \frac{1}{|\alpha|^{1/2}} \int_{-\infty}^{\infty} f(t) \tilde{\psi}\left(\frac{t-\beta}{\alpha}\right) dt,$$

where  $\tilde{\psi}$  is the dual of  $\psi$  in the sense that

$$\langle \tilde{\psi}_{j,k}, \tilde{\psi}_{\ell,m} \rangle = \delta_{j,\ell} \delta_{k,m}.$$

From (3), IWT can be interpreted as the projection of the signal  $f(t)$  onto the wavelet space spanned by the translates of the basic wavelet  $\tilde{\psi}$  scaled by a factor of  $\alpha$ . When  $\alpha$  is set to be  $2^{-k}$  and  $\beta, j/2^k$ , the wavelet series in (1) may be used to compute the coefficients

$$(4) \quad d_{j,k} = (W_{\psi} f) \left( \frac{j}{2^k}, \frac{1}{2^k} \right).$$

One advantage in using wavelet analysis to detect the arrival-times of signals is the variable window size offered by the wavelet. The basic wavelet  $\psi$  in (3) may be regarded as a window function whose width is flexibly controlled by the scale value  $\alpha$  to produce the zoom-in and zoom-out effects used for examining a signal. If  $\alpha$  is chosen to be large, the time-window is wide and the IWT produces good low-frequency resolution. Conversely, a small value of  $\alpha$  generates a narrow time-window for time-domain indication of the arrival-time of a signal.

### 3. Spline-wavelets and algorithms

The compactly supported spline-wave-lets developed by Chui and Wang [2] are explicitly represented by a linear combination of the  $B$ -spline functions. If  $N_m(t)$  denotes the cardinal  $B$ -spline of order  $n$ , then the  $m^{\text{th}}$  order compact supported spline wavelet is given by

$$\psi_m(t) = \frac{1}{2^{m-1}} \sum_{j=0}^{2m-2} (-1)^j N_{2m}(j+1)$$

$$(5) \quad N_2^{(m)}(2t - j).$$

The cubic ( $m = 4$ ) spline-wavelet and its Fourier transform are illustrated in Figures 1 and 2, while the signal processing properties of the spline-wavelets for  $m = 2$  and 4 have been evaluated and tabulated in Tables 1 and 2. It can be seen from Table 1 that the area of the rectangle ( $\Delta\psi_t \times \Delta\psi_\omega$ ) on the time-frequency plane is approximately equal to 0.5 for the cubic spline-wavelet which is the lower limit due to the uncertainty principle. Numerically, we can show that as the order of the spline wavelet is increased, the area of the rectangle in the time-frequency plane approaches the lower limit. Hence the window property of the spline-wavelets is near optimal.

It is also easy to see from Figure 2 that the spline-wavelet is the impulse response of a band-pass filter. The IWT can be interpreted as a band-pass filtering of the signal  $f(t)$  where the center

frequency and the bandwidth is controlled by the parameter  $\alpha$ . For the cubic spline-wavelet, the first sidelobe of the bandpass filter is better than  $-90$  db.

Furthermore, the geometric symmetry of the spline-wavelets guarantees the linear phase property. Consequently, the spline-wavelets may be regarded as linear phase bandpass FIR filters. Two algorithms have been constructed using spline wavelets with these properties for processing transient signals in real-time.

The first is a real-time algorithm for projecting the sampled signal  $f(n)$  onto the spline space. The goal here is to find the coefficients of a spline series approximating the signal. The projection formula

$$(6) \quad f(t) \cong f_a(t) \\ = \sum_i \left\{ \sum_j \xi_{j+\frac{\pi}{2}-it_m}^{(m)} f(ih_0) \right\} N_m \left( \frac{t}{h} - j \right)$$

with  $h_0 = \ell_m h$ , has been used in [4]. For cubic spline ( $m = 4$ ), the weight coefficients  $\xi_n^{(4)}$  are tabulated in [4].

The second algorithm is to decompose the approximating signal  $f_a(t)$  into several orthogonal components at various resolutions. Assuming that  $f_a(t)$  is represented at the  $k^{\text{th}}$  resolution, (6) can be rewritten into a spline series

$$(7) \quad f_a(t) = \sum_j c_{k,j} N_m \left( \frac{t}{h} - j \right) = f_k(t)$$

which can be decomposed into wavelet series at lower resolutions

$$(8) \quad f_k = g_{k-1} \oplus g_{k-2} \oplus \cdots \oplus g_{k-l} + f_{k-l}$$

where  $L$  is an arbitrary positive integer and  $\oplus$  denotes orthogonal sums. The wavelet series for  $g_n(t)$  is

$$(9) \quad g_p(t) = \sum_j d_{p,j} \tilde{\psi}_{p,n} \left( \frac{t}{2^k h} - n \right),$$

where the wavelet coefficients  $d_{p,j}$  are obtained by a moving average operation

$$(10) \quad d_{p,j} = \sum_n b_{n-2j} c_{p+1,n}$$

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followed by subsampling and the spline coefficients  $c_{p,j}$  are also obtained by a similar operation, namely:

$$(11) \quad c_{p,j} = \sum_n a_{n-2j} c_{p+1,n}.$$

The coefficient sequences  $\{a_k\}$  and  $\{b_k\}$  are tabulated in [1].

#### 4. Numerical examples

A wide variety of physical phenomena can be represented by sinusoidal signals bounded by exponential decay envelopes. A signal consisting of three different frequencies is given by:

$$(12) \quad y(t) = \sum_{i=1}^3 e^{-\lambda_i(t-t_i)} e^{j\omega_i t} u(t-t_i)$$

where  $\lambda_i > 0$  are decay constants of the envelopes;  $t_i$  are the arrival-times; and  $\omega_i$  are the signal frequencies. This type of signals has been studied in [5] using a one-sided exponential window which matches the signal waveforms. We analyze these signals by the cubic spline-wavelet using the two algorithms outlined in Section 3. In our example, the signal is corrupted with Gaussian noise. The times of arrival of the three exponentials are clearly shown in Figures 3b and 3c.

We also use two overlapping modulated sinusoids of the same frequency to simulate a receiver pickup of the direct signal and its echo. The echo amplitude is 5% of the direct signal. Figure 4a displays the received signal (direct + echo) and the echo alone. Figure 4b shows the echo separated from the received signal with its arrival time clearly indicated.

#### 5. Conclusions

We have demonstrated a technique to detect the arrival-times of transient signals using compactly supported spline-wavelets. Unlike other techniques which depend on match filtering or statistical algorithms, our algorithms decompose the signal into various frequency bands to apply thresholding techniques. The results of our numerical experiments agree very well with the designed signals.

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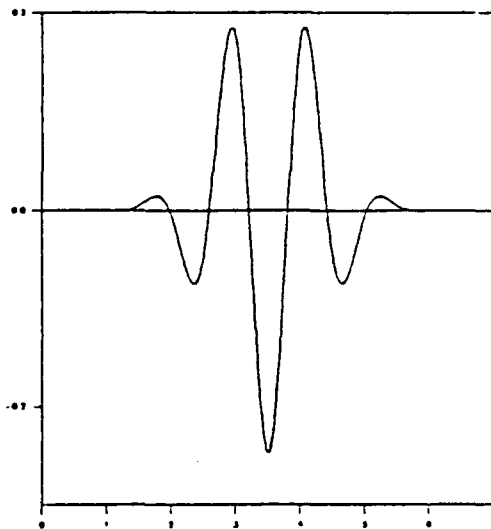


Figure 1 Cubic spline-wavelet  $\psi$

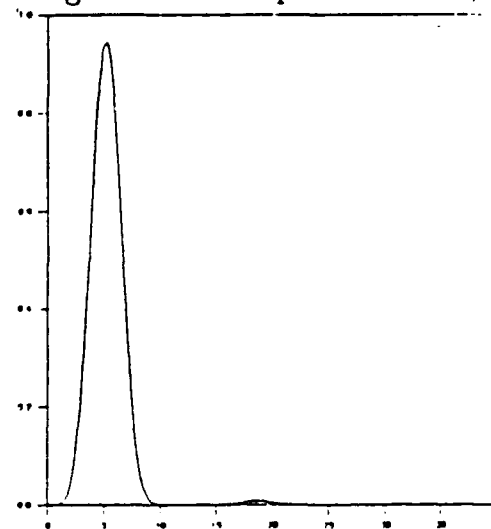


Figure 2 Fourier transform of  $\psi$

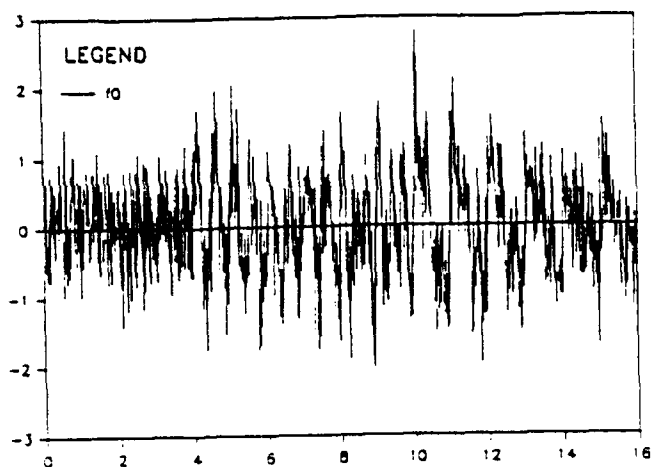


Figure 3a Original signal with noise

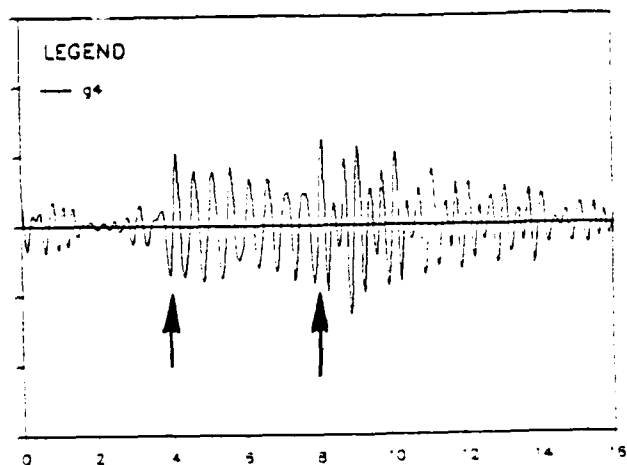


Figure 3b Onsets of the mid- and high frequency signals

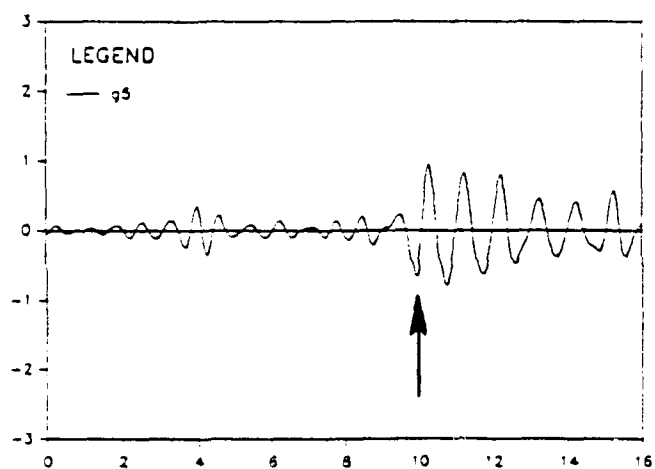


Figure 3c Onset of the low-frequency signal

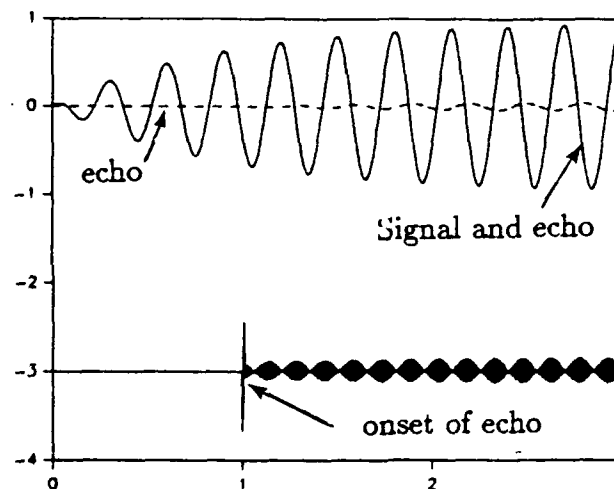


Figure 4

Wavelet Class	Order	$t_0$	$\Delta\psi_t$	$\omega\eta$	$\Delta\psi_\omega$	$\Delta\psi_t + \Delta\psi_\omega$
B-wavelet	m=2	1.5	0.390	5.406	2.493	0.972
	m=4	3.5	0.542	5.164	0.932	0.505
Dual-wavelet	m=2	1.5	0.457	4.778	2.350	1.073
	m=4	3.5	0.873	4.172	1.041	0.909

Table 1: Centers and widths of spline-wavelet windows

m	B-wavelet	Dual-wavelet
2	-45	-53
3	-71	-81
4	-94	-107

Table 2. Out-of-Band rejection in db of the bandpass filter from the wavelets